AE3330 Typed-up notes part 2!!!

SPACE STUFF

Time for space!

Page 139 of the blue book

Space starts at the Karman Line

100 km above sea level.

If you are trying to maintain Lift, you will end up flying out of orbit

There is still drag

KEY METRICS

SOPME MATH!@

Vectors – both magnitude and direction

Some vector review….

Scalar triple product

And vector triple product identities that are useful for some derivations.

They are on page 140.

Kepler and Newton's laws

CONIC SECTIONS

These are types of orbits.

Kepler's first law: each planet moves in an ellipse with the sun at one focus.

Anatomy of an ellipse:

A = semi-major axis

B = semi-minor axis

E = eccentricity

Ra = a(1+e) (the distance between the major body and apoapsis)

Rp = a(1-e) – the distance between the major body and periapsis

This is a Keplerian orbitKplers2nd l-the line drawn from the sun to a planet sweeps out equal areas in equal amounts of time. (diagram on page 141)

2^{nd} law also stated as: angular momentum is constant in an orbit

3^{rd} law:

Ther square of the period of a planet is proportiuonal to the cube of the semi-major axis

Period^2 is proportional to a^3

OK NOW WE ARE IN THE SMALL BLUE COMPOSITION BOOK

Kepler's Laws (in previous notebook)

P = 2pir/v

(period)

4pi^2r^2 / v^2 = kr^3

This is Kepler's 3^{rd} law

V^2 = 4pi^2 / kr

What is k? We can derive this later.

Newton's Laws

Consider a circular orbit

Law of universal gravitiation

-any 2 bodies attract one another with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them.

Law of gracitation:

$F = -GMm/r^2$ (page 002)

Vector form =is on page 2

Standard value for G:

$G = 6.7408 \times 10^{-11}\ Nm^2/kg^2$

Look up the experiment that Cavendish did to determine G

In orbital mechanics

Mu = GM _> this is a gravitational parameter. [mu] = Nm^2 / kg

So F = -mum/r^2 r-hat

We have the N-body problem

(a picture is on page 3)

Fgi = -Gm1mi/r^21i * r-hate

The total gravitational sum is on page 3

This is assuming an inertial frame, or a non-accelerating reference frame

Equation for r-i-double-dot is on page 4

This is the acceleration of mi, and governs the motion of mass i.

Example with m1 as the earth, m2 as a satellite, m3 as the moon, and m4 as the sun is on page 5.

At the bottom of page 5, there is the equation of relative motion. This assumes the central body does not move. And is much larger than the orbiting body.

Mu_earth = $3.986\times10^5\ km^3/s^2$

Mu_sun = $1.327\times10^{11}\ km^3/s^2$

Mu_mars = $4.305\times10^4\ km^3/s^2$.

$M1 >> m2$ is generally a pretty god assumption.

Orbital MOTION

You can think of orbital motion as an exchange of potential and kinetic energy that is constant.

(cannonball analogy)

K. Energy = $1/2mv^2$

….derivation, etc

On page 6.

ENERGY OF ORBIT:

Epsilon = $v^2/2 - mu/r$. (boxed equation at the bottom of page 6)

SPECIFIC ANGULAR MOMENTUM

$H = rxv$

A diagram of gamma, the zenith angle and phi, the flight path angle is on page 7.

H = rvsingamma = rvcosphi.

BREAK

If energy is conserve,

Denergy/dt = 0.

Derivation////derivation/// (page 7 to 8…

We are proving that angular momentum is also conserved.

Let's solve for some orbital motion

Derivation is long.

It's on pages 8 and 9.

Anyway, we get that $r = h^2/mu$ / (1+B/mu * cosnu)

This I on page 9.

No terms are vectors in this equation..

B is the initial vector.

Diagram of elliptical orbit defining nu and periapsis, etc on page 10./

Nu is definted wrt central body/focus.

R = P / (1 + ecosnu) P is the semi-lattice rectum and it is the distance between the focus point and the edge of the ellipse.

You can also get that P = h^2/mu and e = B/mu

So

The equation for r is a;so

R = h^2/mu / (1 + B/mu * cosnu) (just like before.

SOME 2-BODY GEOMETRY TERMS

P = semi-lattice rectum

E = eccentricity, and points in the same direction as B (determines the type of conic section

E = 0 -> circle!

0 < e < 1 -> an ellipse!

E = 1 -> parabola! V_inf = 0

e>1 -> hyperbolda! V_inf > 0

r,v change throughout orbit

h = rvcosphi

also changes throughout orbit

phi = 0 at periapsis and apoapsis.

Peroperty:

- ➔ Ratio of r and given line directrix is constant e.

Circl

R = a = P = b

E =0

Ellipse

2b is distance of shoerter side

2a is distance of longer side

2p is le ngth at focus point

2c is distance between foci.

$0<e<1$

Parabolda

A,c, = infinity (undefinied)

Energy at infinity = 0

E = 1

2P is the width of the parabola at the focus point

Hyperbola

2c is distance between foci

-2a is distance between each part of the hyperbola

$e>1$

2P is width of each hyperbola part at the focus point.

Energy at infinity is greater than 0 (diagrams of all these are on page 11)****

Some relations:

E = c/aw (not true for parabola

$P = a(1-e^2)$

Again not true for parabola b/c a is undefine.

Rp = periapsis at nu = 0

Ra = apoapsis at nu = 180

R = P/(1 + ecosnu

Rp = P(1+e) = a(1-e)

Ra = P/(1-e) = a(1+e)

H = rvcosphi

H = rpvp = rava (in this case, phi = 0, cosphi = 1)

Pg12

ENERGY

E = vp^2/2 – mu/rp

At any orbit

E (energy) = -mu/2a.

This is negative when you are “in the gravity well”

-positive or zero when you can escape it.

Elliptical orbits

Epsilon (energy for orbits) = -mu/2a is valid for all conic sections

A summary table of geometry is on page 14.

Let’s dive into these in more detail!!

Starting with Elliptical orbit

One bood, two nipples.

Sum of distances from any point on the sllipse to each focus is constant.

Ellipse geometry…

R + r’ = 2a

Rp + ra = 2a

Ra-rp = 2c

A^2 = b^2 + c^2

E = c/a = ra-rp / ra + rp

Orbital period for an ellipse:

RP (PERIOD) = 2pia^3/2 / sqrtmu)

Derivation on page 15.

Period for a circular orbit:

E = 0

R = p = a

Ra, rp = r

TP = 2pir^3/2 / sqrt(mu)

Veclocity for a circular orbit:

V_c = sqrt (mu.r)

As r increases, vc decreases and period increases.

For a circular orbit, phi is always 0, v is always perpendicular to r.

H = rvcosphi = sqrt Pmu)

Rvcosphi = sqrt(rmu)

Etc.

Parabolic orbits

PE = KE

R_inf changes less and less the further out you go.

Escape speed -> velocity at some radius r such that the orbiting body will reach r_inf at v_inf = 0. Wrt to central body

$E = v^2/2 – mu/r$ = constant for 2-body.

$V^2escape / 2 = mu/r$

$V_{esc} = sqrt (2mu/r)$

Compare with circular velocity – there is a difference of factor sqrt 2

You can achieve V_esc wrt any radius, doesn't have to be rp. As r increases, v_esc decresases.

Hyperbolic orbits

(a diagram of hyperbolae is very detailed and on page 18)

$C^2 = a^2 + b^2$

Delta = "turn angle," which is the angle between asymptotes sometimes called "bend angle" or "flyby angel"

Sindelta/2 = a/c

E = c/a

Din(delta/2) = 1/e

As e increases, delta decreases.

As r goes to infinite, v goes to v_inf, which is greater than 0..

V_inf -> hyperbolic excess speed -> amount of speed you have left at the end of your trajectory.

Epsilon = vbo^2/2 – mu/rbo = v_inf^2 / 2

Bo = "burn out"

Can solve for v_inf = [Vbo^2 – 2mu/rbo] ^1/2

(this is on page 18)

What is burnout?

Rbo = position on your priginal orbit where you perform burn (may or may not be apoapsis)

Vbo is velocity at rbo after burn

Decide to add energy to system.

Can consider delta-v to be instantaneous.

Vbo = v + deltav

If you add veloity, the rbo point becomes rp.

If you take away velocity, rbo becomes ra.

Sphere of Influence (SOI)

Reality: not everything is a 2-body problem

-an arbitrary line , but an important one nonthe less

-we say a spacecraft has escaped when v _. V_inf, but also when Fg -> 0 (for all practical purposes.)

Fg is proportional to 1/r^2 so F_g decreaseas as r increases.

R_SOI = D (m2/m1) ^2/5

D = mean distance

M1 = mass of central body

For a planet like Earth, it's possible to be in multiple SOI

(equation for SOI sphere of influence on page 19 and diagram too)

Because satellites are n both SOI of moon and earth, it's very difficult to estimate lunar orbiting as 2-body problem

R_SOI = 66,300km wrt Earth

Orbit DETERMINATION

Canonical units

-we have this normal system of units that's anything but normal.

1 mass unit: = 1 MU mass of sun

1 distance = 1 DU = mean radius of Earth's orbit around Sun

1 time unit : 1 TU = such that Earths's orbital speed around the Sun is 1 DU/TU

Mu = 1AU^3 / TU^2 (Sun)

1 AU = 1/486x10^8 km (DU)

1 TU = 1AU/V_E (mean velocity of Earth)

OTHER THAN THE SUN

1 MU = mass of primary body

1 DU = radius of primary body

1 TU = time unit such that orbital velocity of somebody or spacecraft "skimming" the planet's surface in (at height zero, basically)

Picked such that orbital velocity 1 DU/TU

1

This is confusing and it's on page 21.

Kepler's Problem

dA/dt = constant

Position and velocity are a function of time

(t-T)

Time it took from periapsis

(t-T)/A1 = TP/phab

This is the ratip of total area to period = ratio of A1 to time to sweep A1.

If we solve for A1, we can solve for t.

This equation is on page 21.

Derivation and pictures on pages 22 and 23

Bottom line…

A1 = ab/2 [E – esinE]

Also

(t-T) = sqrt(a^3/mu) * (E – esinE)

T is the time since passing periapsis.

Kepler introduced another measure:

M = E – esinE (mean anomaly

N = sqrt mu/a^3) = 2 pi / TP

So…

(t-T) = m/n.

This is Kepler's equation

$M = n(t-T) = E – esinE$

To make it useful. You need to find a way to relate E to nu.

cosE = (e + cosnu / (1 + ecosnu)

AND

Cosnu = (cosE-e) / 1 + ecosE)

(eq's 4.5 and 4.6 on page 24)

Characteristics of Em, nu, and M….

1) Defined as between 0 and 2pi
2) 2) when one is 0, all are 0
3) 3) when on is pi, all l are pi
4) 4) all 3 are the same half plane wrt pi
5) 5) when e = 0. E = nu

Calculate time of flight:

We have two sitatuions and two equations, summarized in a third equation.

Look at page 25 for that.

Calculating time of flight for pararbolas and hyperbolas are equations 6 and 7 on pa 26.

Kepler's inverse problem

It's tough to solve kepler's problem for E.

1) Select E
2) 2) find M€

3) 3) adjust E based on M(eE) from time
4) 4) Iterate
5) (however, you get numerical issues when you do this)

OK

STILLL on the subject of orbit determination, we're going to start talking about orbital determination with coordinate systems

The cohisce is problem-dependent

1) Coordinate frame
2) -> reference frame
3) -> reference direction
4) -> reference time
   (epoch)
   (currently we used J2000, based on Jan 1st 200, 12 midnight EST)

- ➔ Reference body (sun, earth, moon, etc)
- ➔ -> location of origin

2) define center of coordinate frame

->z-axis

3) Type

-> cartesian, spherical, etc.

COMMON COORDINATE SYSTEMS

1) Heliocentric-Ecliptic
2) (cartesian, inertial)
3) Ref body: sun
4) Ref plane: earth ecliptic
5) Ref direction +x vernal equinox
6) Page 27
7) 2)
8) Planet-centric equatorial (cartesian or spherical)
9) -inertial
10) -sometimes called ECI
11) (earth-centered inertial)

Ref body -> Earth's origin or CM

Ref plane _. Equatorial plane

Ref direction -> +x vernal equinpox

Z -> through axis of rotation

+z northern hemisphere

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3) international celestial reference system

-barycenter of solar system

-spherical coord

Alpha = angle from x in ref plane

Delta = angle from ref plane to r vector.

5) Topo-centric coordinate system
6) Ref body = earth (fixed to surface)
7) Ref plane = local horizon
8) Ref direction = South
9) MOST IMPORTANT, ALONG W/ ECI:
10) 5) perifocial eccentricity (cartesian)
11) Ref body = earth or another
12) Ref plane = orbital plane
13) Ref direction = periapsuis
14) Yq w in direction of motion
    90 degrees from xq
    Zw direction of h (specific angular momentum)
    For a CIRCLE, reference direction points toward intersection of orbital and equatorial plane
    -for a circle in equatorial orbit, ref direction is vernal equinox.

6 Classical ORIBTAL ELEMENTS

For any orbit, you need 6 pieces of information to know what's going on.

Btw diarams of these are all on pages 27 - 29

DIAGRAM on page 30

1: a, semimajor axis: defines size of orbit

(P used sometimes)

2: e: eccentricity -> determines type of orbit

3: i: inclincation: angle between z and h/zw

4: big omega: longitude of ascending notes (right ascension of the ascending noted)

Angle in the xy plane between x axis and the position where s/c or orbiting body crosses the equatorial plane while moving in the +z direction measured CCW when viewed from +z down on xy plane

5: small omega: argument of perigee

Angle in the plane of S/C orbit between ascending node of periapsis measured in diection of S/C motion

6: nu: true anomaly

Angle in the xq,yw plane between periapsis and current S/C position.

OTHER ORBITAL ELEMENTS

7: big PI : longitude of periapsis.

PI = big omega plus small omega.

8: sub for nu:

Tp : time of periapsis passage (can be in the past)

9: U: argument of latitude

->< angle between ascending node and position of SC ® on xwyw plane

U = nu + omega) this is more convenient for circular orbits

10: l = true longitude

Angle between x and r, measured eastward from ascending node, then in orbital plane to r.

L =- w + nu + big omega

NAMES FOR ORBITS

Posigrade orbit (direct orbit)

-moves in the direction of primary body rotation

0 < I < 90

Retrograde orbit

➔ Moves opposite primary body
Rotation
90 < I < 180
For equatorial orbits
We only need 5 elements
A, e, i=0, pi, and nu
For cicular orbits, we only need 5 elements:
A, I, big omega, e=0, and U
For circular equatorial orbits, we only need 4 elements:
A,e = 0 ,I, l

Page 33
Starts a new day!
Finding orbital elements from r,v!
R,v -> 6 pieces of information
Planet-centered equatorial system

STOPPED ON PAGES 33 and 34

$$R_{11} = \mathbf{I} \cdot \mathbf{P} = \cos\Omega\cos\omega + \sin\Omega\sin\omega\cos(\boldsymbol{\pi}\text{-}i)$$

$$= \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i\,.$$

Similarly, the elements of $\widetilde{\mathbf{R}}$ are

$$R_{11} = \cos\Omega\cos\omega - \sin\Omega\sin\omega\cos i$$

$$R_{12} = -\cos\Omega\sin\omega - \sin\Omega\cos\omega\cos i$$

$$R_{13} = \sin\Omega\sin i$$

$$R_{21} = \sin\Omega\cos\omega + \cos\Omega\sin\omega\cos i$$

$$R_{22} = -\sin\Omega\sin\omega + \cos\Omega\cos\omega\cos i$$

$$R_{23} = -\cos\Omega\sin i$$

$$R_{31} = \sin\omega\sin i$$

$$R_{32} = \cos\omega\sin i$$

$$R_{33} = \cos i$$